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Original Research Article

Stability Analysis of a System of First-Order Linear Sylvester Differential Equations

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Abstract:
Using the new concept of bounded solutions, we establish the stability criterion of linear Sylvester matrix differential equations, and deduce the existence of bounded solutions in ψ with a special case.

Introduction

In this paper, we shall be concerned with the Sylvester system of first order linear differential non-homogeneous equation and establish a necessary and sufficient condition for the existence of (Φ,Ψ)bounded solutions and deduce the results of Lyapunov systems as a particular case. We establish variation of parameters formula and use it as a tool to establish our main results. Sylvester system of first order linear non-homogeneous equation is an interesting area of current research and the general form of its solution in two fundamental matrices is only established by Murty and Prasad in the year of 1989 [9]. The paper attracted many eminent mathematicians like Richard Bellman, Don Fausett, Lakshmikantham to mention a few. Recent results established byViswanadh, V. Kanuri, et. al., is the main motivation behind our results. The concept of Ψ-bounded solutions for linear system of differential equations is due to T. G. Halam [14]. The variation of parameters formula we established is new and will have significant contributions on control engineering problems. The novel idea adopted by Viswanadh, Wu and Murty [8] on the existence of (Φ⨂Ψ)bounded solutions and on the existence of Ψ-bounded solutions by Kasi Viswanadh, V. Kanuri, et. al. [4-7,11,12,13] on time scale dynamical systems is a useful and significant contribution to the theory and differential and difference equations. Further these ideas have been extended by Kasi Viswanadh V. Kanuri to fuzzy differential equations in a novel concept, and is very interesting and useful contribution to the theory of differential equations and also in applications to control systems. The results established on stability, controllability criteria established on state scale dynamical systems on first order linear systems [9] can be generalized to (Φ,Ψ)bounded solutions to Sylvester linear system of differential equations. This paper is organized as follows: section 2 presents a criterion for the existence of Φ-bounded solution of the matrix linear system $T'=AT$and Ψ-bounded solution of the linear system $T'=B*T$(where * refers to the transpose of the complex conjugate). By super imposing these two solutions, we establish the general solution of the linear matrix Sylvester system$T'=A(t)T+TB(t)$(1.1) where Tis a square matrix of order (n ×n) and $A(t)$, $B(t)$are also n ×n matrices. We present our basic results that are available in literature [4, 5, 6, 7, 8, 10, 11]. Our main results are established in section 3. This section also presents criteria for the Sylvester system (1.1) to be stable, asymptotically stable, and establishing the result on controllability. Throughout this paper, (t)stands for a fundamental matrix solution of the linear system.

$T'=(t)T$(1.2) and $Z(t)$stands for a fundamental matrix solution of the linear system$T'=B*T$. (1.3) 2. Preliminaries In this section, we shall be concerned with establishing general solution of the Sylvester linear system and present ΦY-bounded solution of the linear system (1.2) and then ΨZ-bounded solution of the system (1.3).Theorem 2.1 Tis a solution of (1.1) if and only if $T=YCZ*$, where Cis a constant square matrix and Y is a fundamental matrix solution of (1.2) and Z is a fundamental matrix solution of (1.3).Proof: It can easily be verified that Tdefined by $YCZ*$is a solution of (1.1). For$T'=Y'CZ*+YCZ*'=(t)YCZ*+YCZ*B=A T+TB$.Hence, $YCZ*$is a solution of (1.1). Now, to prove that every solution is of this form, let Tbe a solution, and Kbe a matrix defined by $K=Y-1T$. Then, $Y'K+YK'=AYK+YKB$*or $YK'=YKB$*or $K'=KB$*or K*' = BK*. Since $Z$is a fundamental matrix solution of (1.3), it follows that there exists a constant square matrix Csuch that $K*=ZC*$or $K=CZ*$. Since $T=YK=YCZ*$. □In [4], Kasi Viswanadh, V. Kanuri, et. al.presented a novel concept on Ψ-bounded solutions of linear differential systems on time scales. We use these ideas as a tool to establish (Φ,Ψ)bounded solutions of the Sylvester system (2.1). If B is replaced by A*, we get Lyapunov system. In this case the general solution is given by YCY*.Definition 2.1 A function$Y$:ℝ+⟶ℝn2is said to be Φ-bounded solution onℝif Φ$Y$is bounded on ℝ.Definition 2.2 A function Z:ℝ+⟶ℝn2is said to be Ψ-bounded solution onℝif $Z*\Psi*$is bounded on ℝ.Definition 2.3 A function Y:ℝ+⟶ℝn2is said to be Φ-Lebesgue integrable on ℝ+if $Y(t)$is measurable and $\Phi(t)Y(t)$is Lebesgue integrable on ℝ+.Definition 2.4 A function Z:ℝ+⟶ℝn2is said to be Ψ-Lebesgue integrable on ℝ+if $Z*(t)$is measurable and $Z*(t)\Psi*(t)$is Lebesgue integrable on ℝ+.Let Φi:ℝ+⟶ℝn, i=1,2,...,n, be continuous and let Φ(t)=(Φ1(t),Φ2(t),...,Φn(t))be linearly independent so that Φis invertible and also we assumeΨis invertible.By a solution of the linear system (1.1), we mean $Y(t)CZ*(t)$, which is an absolutely continuous function and satisfies (1.1) for almost all t≥0.Let Ybe a fundamental matrix solution of (1.2) satisfying $Y(0)=In$and Zbe a fundamental matrix solution of (1.3) satisfying$Z(0)=In$. Let $X1$denote the subspace of ℝ$n$consisting of all vectors whose values are of Φ-bounded solutions of (1.2) for t=0and $X2$be the arbitrary fixed subspace of ℝ$n$supplementary to$X1$. Further, let $P1$be the projection matrix of ℝ$n$onto $X1$($P12=P1$and $P1$:ℝn⟶ℝn) and let $P2=I-P1$be the projection matrix on $X2$Definition 2.5 A function f:ℝ⟶ℝ$n\times n$is said to be Φ-bounded on ℝif $\Phi(t)f(t)$is bounded on ℝ, i.e. sup$t$∈ℝ‖$\Phi(t)f(t)$‖<∞.Definition 2.6 A matrix Y:ℝ⟶ℝ$n\times n$is saidto be Φ-bounded on ℝif the matrix $\Phi(t)Y(t)$is bounded on ℝ, i.e. there exists an M>0such that sup$t$∈R‖$\Phi(t)Y(t)$‖≤MDefinition 2.7 A matrix Y:ℝ⟶ℝ$n\times n$is said to be Φ-integrable on ℝcomponent-wise if $\Phi(t)Y(t)$is integrable on ℝ, i.e.∫‖$\Phi(t)Y(t)$‖dt<∞∞0.Definition 2.8 A matrix function (Φ,Ψ):ℝ⟶ℝ$n2\times n2$is said to be (Φ,Ψ)-bounded if the matrix‖Φ$YZ*\Psi*$‖ is bounded on ℝ.

3. Main results In this section, we shall be concerned with the existence of (Φ,Ψ)-bounded solution of the linear Sylvester system (1.1), and then present the stability and asymptotic stability of the Sylvester system.Theorem 3.1 Let Aand Bbe $(n\times n)$ continuous square matrices on ℝ. Then, the system (1.1) has at least one (Φ,Ψ)-bounded solution on ℝfor every continuous (Φ,Ψ)-bounded function if and only if there exists a positive constant Ksuch that ∫‖Φ(𝑡)𝑌(𝑡)𝑃𝑍∗(𝑡)Ψ∗(𝑡)‖∞−∞≤𝐾for all 𝑡≥0(3.1)where 𝑃=𝑃−on (−∞,𝑡),𝑃=𝑃0+𝑃+on (𝑡,0),𝑃=𝑃+on (0,∞), 𝑃=𝑃−on (−∞,0), 𝑃=𝑃0+𝑃−on (0,𝑡), 𝑃=𝑃+on (𝑡,∞).Proof: First, suppose the linear Sylvester system has at least one (Φ,Ψ)-bounded solution on ℝfor every continuous (Φ,Ψ)-bounded function on ℝ. Then, it is claimed that there exists a constant 𝐾>0such that the inequality (3.1) holds. Let Bbe the Banach space of all (Φ,Ψ)-bounded continuous functions 𝑇:ℝ→ℝ𝑛2with norm ‖𝑇‖B=sup𝑡∈ℝ‖Φ(𝑡)𝑌(𝑡)𝑍∗(𝑡)Ψ∗(𝑡)𝑇(𝑡)‖, we define(i) C: the Banach space of all (Φ,Ψ)-bounded continuous functions 𝑇:ℝ→ℝ𝑛2with norm‖𝑇‖𝐶=sup𝑡∈ℝ‖𝛷(𝑡)𝑌(𝑡)𝑍∗(𝑡)𝛹∗(𝑡)‖(ii) B: the Banach space of all (Φ,Ψ)−∆integrable functions 𝑇:ℝ→ℝ𝑛2with norm

||T||B =∫‖𝛷(𝑡)𝑌(𝑡)𝑍∗(𝑡)𝛹∗(𝑡)‖𝑑𝑡∞−∞(iii) D: the set of all continuous function 𝑇:ℝ→ℝ𝑛2which are absolutely continuous on all intervals 𝐽⊂ℝ, (Φ,Ψ)-bounded on ℝ, 𝑇(0)∈𝑋−⊗𝑋+, and**𝑻′**=**𝑨𝑻**+**𝑻𝑩**∈B. Step 1: We first claim that (𝐷,‖‖𝐷)is a Banach space. For, we first note that (𝐷,‖‖𝐷)is a vector space. Let {𝑇𝑛}∈ℕbe a fundamental sequence in B. Then, there exists a continuous (Φ,Ψ)-bounded function on ℝsuch thatlim𝑛→∞Φ𝑛(𝑡)𝑇𝑛(𝑡)Ψ𝑛∗(𝑡)𝑇𝑛∗(𝑡)=Φ(𝑡)𝑇(𝑡)Ψ∗(𝑡)𝑇∗(𝑡)Uniformly onℝ. From the inequality‖𝑇𝑛(𝑡)−𝑇(𝑡)‖≤‖𝛷−1(𝑡)‖{‖𝛷(𝑡)𝛹∗(𝑡)𝛹∗−1(𝑡)‖‖𝛹∗(𝑡)‖−‖𝛷−1(𝑡)‖‖𝛷(𝑡)𝑇(𝑡)𝛹∗(𝑡)‖‖𝛹∗−1(𝑡)‖}Hence,

lim𝑛→∞𝑇𝑛(𝑡)=𝑇(𝑡)uniformly on every compact subset of ℝ. Thus,(0)∈𝑋−⊗𝑋+. Thus, (𝐷,‖‖𝐷)is a Banach space.We now establish variation of parameters formula for the non-homogeneous Sylvester system𝑇′=𝐴(𝑡)𝑇+𝑇𝐵(𝑡)+𝐹(𝑡)(3.2)where 𝐹(𝑡)is a given $(n\times n)$ square matrix. Let Tbe any solution of (3.2) and 𝑇be a particular solution of (3.2). Then 𝑇−𝑇is a solution of the homogenous system (1.1). Any solution of the homogeneous system is of the form 𝑇(𝑡)=𝑌(𝑡)𝐶𝑍∗(𝑡), where 𝑌(𝑡)is a fundamental matrix solution of (1.2) and 𝑍(𝑡)is a fundamental matrix of (1.3). Such a solution cannot be a solution of (3.1) unless(𝑡)=0.We seek a particular solution of (3.1) in the form𝑇(𝑡)=𝑌(𝑡)𝐶(𝑡)𝑍∗(𝑡)

and see that 𝑇(𝑡)is a particular solution of (3.1).
Now,𝑇′(𝑡)=𝑌′(𝑡)𝐶(𝑡)𝑍∗(𝑡)+𝑌(𝑡)𝐶′(𝑡)𝑍∗(𝑡)+𝑌(𝑡)𝐶(𝑡)𝑍∗′(𝑡)=𝐴(𝑡)𝑌(𝑡)𝐶(𝑡)𝑍∗(𝑡)+𝑌(𝑡)𝐶′(𝑡)𝑍∗(𝑡)+𝑌(𝑡)𝐶(𝑡)𝑍∗𝐵

SriRam Bhagavatula,IJECS Volume 09Issue 11November, 2020 Page No.25252-25259 Page 25255Now on substitution in the eqn (3.1) gives 𝐴(𝑡)𝑌(𝑡)𝐶(𝑡)𝑍∗(𝑡)+𝑌(𝑡)𝐶′(𝑡)𝑍∗(𝑡)+𝑌(𝑡)𝐶(𝑡)𝑍∗𝐵=𝐴(𝑡)𝑌(𝑡)𝐶(𝑡)𝑍∗(𝑡)+𝑌(𝑡)𝐶(𝑡)𝑍∗(𝑡)𝐵(𝑡)+𝐹(𝑡)which gives 𝑌(𝑡)𝐶′(𝑡)𝑍∗(𝑡)=𝐹(𝑡), then 𝐶′(𝑡)=𝑌−1(𝑡)𝐹(𝑡)𝑍∗−1(𝑡), or𝐶(𝑡)=∫𝑌−1(𝑠)𝐹(𝑠)𝑍∗−1(𝑠)𝑡𝑎𝑑𝑠and hence, 𝑇(𝑡)=𝑌(𝑡)∫𝑌−1(𝑠)𝐹(𝑠)𝑍∗−1(𝑠)𝑡𝑎𝑑𝑠𝑍∗(𝑡). Now, it can easily be verified that 𝑇(𝑡)is a solution of (3.2), and the general solution is given by𝑇(𝑡)=𝑌(𝑡)𝐶𝑍∗(𝑡)+𝑇(𝑡)=𝑌(𝑡)𝐶𝑍∗(𝑡)+𝑌(𝑡)∫𝑌−1(𝑠)𝐹(𝑠)𝑍∗−1(𝑠)𝑡𝑎𝑑𝑠𝑍∗(𝑡).We now claim that three exists a constant 𝐾0>0such that for every 𝐹∈B and for corresponding solution of 𝑇∈𝐷of (3.2), we havesup𝑡∈ℝ‖Φ(𝑡)𝑇(𝑡)Ψ∗(𝑡)‖≤𝐾0sup𝑡∈ℝ‖Φ(𝑡)𝐹(𝑡)Ψ∗(𝑡)‖or sup𝑡∈ℝmax1≤𝑖≤𝑛‖Φ𝑖(𝑡)𝑇𝑖(𝑡)Ψ𝑖∗(𝑡)‖≤𝐾0sup𝑡∈ℝmax1≤𝑖≤𝑛‖Φ𝑖(𝑡)𝐹𝑖(𝑡)Ψ𝑖∗(𝑡)‖or sup𝑡∈ℝmax1≤𝑖≤𝑛1≤𝑗≤𝑛‖Φ𝑖𝑗(𝑡)𝑇𝑖𝑗(𝑡)Ψ𝑖𝑗∗(𝑡)‖≤𝐾0sup𝑡∈ℝmax1≤𝑖≤𝑛1≤𝑗≤𝑛‖Φ𝑖𝑗(𝑡)𝐹𝑖𝑗(

𝑡)Ψ𝑖𝑗∗(𝑡)‖(3.3)For, define the mapping 𝑅:𝐷→𝐵as 𝑅𝑇=𝑇′−𝐴𝑇−𝑇𝐵. Clearly, Ris linear and bounded with ‖𝑅‖≤1. Let 𝑅𝑇=0, and the fact T satisfies the differential equation𝑇′=𝐴𝑇+𝑇𝐵and hence 𝑇∈𝐵. This shows that Tis (Φ,Ψ)-bounded onℝof the system (1.1). Then(0)∈𝑋0∩(𝑋−⨁𝑋+)={0}.Thus, 𝑇=0so that Ris one-to-one. To prove that Ris “onto”, for any 𝐹∈B, let Tbe a (Φ,Ψ)-bounded onℝof the system (1.1) and Tbe the solution of the Cauchy problem𝑇′=𝐴(𝑡)𝑇+𝑇𝐵(𝑡)+𝐹(𝑡)satisfying 𝑇(0)=(𝑃−+𝑃+)𝑇(0). Then, 𝑈=𝑅−𝑇is a solution of the system (3.2) with (0)=𝑅(0)−(𝑃−+𝑃+)𝑇(0). Thus, 𝑈∈𝐷and 𝑅𝑇=𝐹. Consequently, the mapping Ris a bounded, one-to-one linear operator from one Banach space 𝐵to another Banach space B . Hence, 𝑅−1exists and bounded, where‖𝑅−1𝐹‖𝐵≤‖𝑅−1‖‖𝐹‖𝐵for all 𝐹∈B It follows that ‖𝑅−1𝐹‖=(‖𝑅−1‖−1)‖𝐹‖≤𝐾0‖𝐹‖𝐵where𝐾0=‖𝑅−1‖−1, which is equivalent to (3.3).Let 𝜃1and 𝜃2be any fixed real numbers such that 𝜃1<0<𝜃2and 𝐹:ℝ→ℝ𝑛2be a function in B which vanishes on (−∞,𝜃1]∪[𝜃2,∞). Then it is easy to see that the function 𝑇:ℝ→ℝ𝑛2defined as

SriRam Bhagavatula,IJECS Volume 09Issue 11November, 2020 Page No.25252-25259 Page 25256𝑇(𝑡)={−∫0𝜃1𝜙(𝑡)𝑃0𝜙−1(𝜎(𝑠))𝑓(𝑠)𝑑𝑠 −∫𝜃2𝜃1𝜙(𝑡)𝑃+𝜙−1(𝜎(𝑠))𝑓(𝑠)∆𝑠,𝑡<𝜃1−∫𝑡𝜃1𝜙(𝑡)𝑃−𝜙−1(𝜎(𝑠))𝑓(𝑠)∆𝑠+∫𝑡0𝜙(𝑡)𝑃0𝜙−1(𝜎(𝑠))𝑓(𝑠)∆𝑠−∫𝜃2𝜃1𝜙(𝑡)𝑃−𝜙−1(𝜎(𝑠))𝑓(𝑠)∆𝑠,,𝜃1≤𝑡≤𝜃2∫𝜃20𝜙(𝑡)𝑃+𝜙−1(𝜎(𝑠))𝑓(𝑠)∆𝑠,𝑡>𝜃2is the solution in D of the system (1.1). Now if we put𝐺(𝑡,𝑠)={Φ(𝑡)Ψ∗(𝑡)𝑃−Ψ∗−1(𝑠)Φ−1(𝑠),𝑠≤0<𝑡Φ(𝑡)Ψ∗(𝑡)(𝑃0+𝑃+)Ψ∗−1(𝑠)Φ−1(𝑠),0<𝑠<𝑡−Φ(𝑡)Ψ∗(𝑡)𝑃+Ψ∗−1(𝑠)Φ−1(𝑠),0<𝑡≤𝑠Φ(𝑡)Ψ∗(𝑡)𝑃−Ψ∗−1(𝑠)Φ−1(𝑠),𝑠<𝑡≤0−Φ(𝑡)Ψ∗(𝑡)(𝑃0+𝑃+)Ψ∗−1(𝑠)Φ−1(𝑠),𝑡≤𝑠<0−Φ(𝑡)Ψ∗(𝑡)𝑃+Ψ∗−1(𝑠)Φ−1(𝑠),𝑡≤0≤𝑠.Then, Gis continuous on ℝ𝑛2at all points except at 𝑡=𝑠, and at 𝑡=𝑠Ghas a jump discontinuity of unit-magnitude (In). Then, we have 𝑇(𝑡)=∫𝐺(𝑡,𝑠)𝐹(𝑠)𝑑𝑠𝜃2𝜃1for 𝑡∈ℝ.Indeed, for 𝜃1>𝑡, we have ∫𝐺(𝑡,𝑠)𝐹(𝑠)𝑑𝑠𝜃2𝜃1=−∫Φ(𝑡)Ψ∗(𝑡)(𝑃0+𝑃+)Ψ∗−1(𝑠)Φ−1(𝑠)𝐹(𝑠)𝑑𝑠0𝜃1−∫Φ(𝑡)Ψ∗(𝑡)𝑃+Ψ∗−1(𝑠)Φ−1(𝑠)𝐹(𝑠)𝑑𝑠𝜃20.Rewrite the second integral as ∫...𝑑𝑠+𝜃10∫...𝑑𝑠𝜃2𝜃1, we get∫𝐺(𝑡,𝑠)𝐹(𝑠)𝑑𝑠𝜃2𝜃1=−∫Φ(𝑡)Ψ∗(𝑡)𝑃0Ψ∗−1(𝑠)Φ−1(𝑠)𝐹(𝑠)𝑑𝑠0𝜃1−∫Φ(𝑡)Ψ∗(𝑡)𝑃+Ψ∗−1(𝑠)Φ−1(𝑠)𝐹(𝑠)𝑑𝑠𝜃2𝜃1=𝑇(𝑡).For 𝑡∈[𝜃1,0), we have∫𝐺(𝑡,𝑠)𝐹(𝑠)𝑑𝑠𝜃2𝜃1=−∫Φ(𝑡)Ψ∗(𝑡)𝑃−Ψ∗−1(𝑠)Φ−1(𝑠)𝐹(𝑠)𝑑𝑠𝑡𝜃1−

SriRam Bhagavatula,IJECS Volume 09Issue 11November, 2020 Page No.25252-25259 Page 25257∫Φ(𝑡)Ψ∗(𝑡)(𝑃0+𝑃+)Ψ∗−1(𝑠)Φ−1(𝑠)𝐹(𝑠)𝑑𝑠−0𝑡∫Φ(𝑡)Ψ∗(𝑡)𝑃+Ψ∗−1(𝑠)Φ−1(𝑠)𝐹(𝑠)𝑑𝑠𝜃20=∫Φ(𝑡)Ψ∗(𝑡)𝑃−Ψ∗−1(𝑠)Φ−1(𝑠)𝐹(𝑠)𝑑𝑠𝑡𝜃1+∫Φ(𝑡)Ψ∗(𝑡)𝑃0Ψ∗−1(𝑠)Φ−1(𝑠)𝐹(𝑠)𝑑𝑠−𝑡0∫Φ(𝑡)Ψ∗(𝑡)𝑃1Ψ∗−1(𝑠)Φ−1(𝑠)𝐹(𝑠)𝑑𝑠𝜃2𝑡=𝑇(𝑡).For 𝑡∈(0,𝜃2), we have∫𝐺(𝑡,𝑠)𝐹(𝑠)𝑑𝑠𝜃2𝜃1=∫Φ(𝑡)Ψ∗(𝑡)𝑃−Ψ∗−1(𝑠)Φ−1(𝑠)𝐹(𝑠)𝑑𝑠0𝜃1+∫Φ(𝑡)Ψ∗(𝑡)(𝑃0+𝑃−)Ψ∗−1(𝑠)Φ−1(𝑠)𝐹(𝑠)𝑑𝑠−𝑡0∫Φ(𝑡)Ψ∗(𝑡)𝑃+Ψ∗−1(𝑠)Φ−1(𝑠)𝐹(𝑠)𝑑𝑠𝜃2𝑡=∫Φ(𝑡)Ψ∗(𝑡)𝑃−Ψ∗−1(𝑠)Φ−1(𝑠)𝐹(𝑠)𝑑𝑠𝑡𝜃1+∫Φ(𝑡)Ψ∗(𝑡)𝑃0Ψ∗−1(𝑠)Φ−1(𝑠)𝐹(𝑠)𝑑𝑠−𝑡0∫Φ(𝑡)Ψ∗(𝑡)𝑃+Ψ∗−1(𝑠)Φ−1(𝑠)𝐹(𝑠)𝑑𝑠𝜃2𝑡=𝑇(𝑡).For 𝑡>𝜃2, we can easily show that ∫𝐺(𝑡,𝑠)𝐹(𝑠)𝑑𝑠𝜃2𝜃1=𝑇(𝑡). Therefore, sup𝑡∈ℝ‖Φ(𝑡)Ψ∗(𝑡)∫𝐺(𝑡,𝑠)𝐹(𝑠)𝑑𝑠𝜃2𝜃1‖≤𝐾∫‖Φ(𝑡)Ψ∗(𝑡)𝐹(𝑡)‖𝑑𝑡𝜃2𝜃1for all 𝑡∈ℝ. Hence,‖Φ(𝑡)Ψ∗(𝑡)𝐺(𝑡,𝑠)Ψ∗−1(𝑠)Φ−1(𝑠)‖≤𝐾for all 𝑡∈ℝ. Now, to prove the converse statement, suppose the fundamental matrices of 𝑌and 𝑍of (1.2) and (1.3) satisfy the condition (3.1) for some 𝐾>0. Let 𝐹:ℝ→ℝ𝑛2be a Lebesgue (Φ,Ψ)-delta integrable function onℝ. We consider the function 𝑈:ℝ→ℝ𝑛2defined by𝑈(𝑡)=∫Φ(𝑡)Ψ∗(𝑡)Ψ∗−1(𝑠)Φ−1(𝑠)𝑇(𝑠)𝑑𝑠∞−∞+∫Φ(𝑡)Ψ∗(𝑡)Ψ∗−1(𝑠)Φ−1(𝑠)𝑇(𝑠)𝑑𝑠−𝑡0∫Φ(𝑡)Ψ∗(𝑡)Ψ∗−1(𝑠)Φ−1(𝑠)𝑇(𝑠)𝑑𝑠∞𝑡. (3.3)Then, the function is well defined onℝ, and ‖Φ(𝑡)Ψ∗(𝑡)𝑈(𝑡)‖≤𝐾∫‖Φ(𝑠)Ψ∗(𝑠)𝑇(𝑠)‖𝑑𝑠∞−∞,which shows that Uis (Φ,Ψ)-bounded onℝ. Hence the proof is complete. □

SriRam Bhagavatula,IJECS Volume 09Issue 11November, 2020 Page No.25252-25259 Page 25258Theorem 3.2: If the homogeneous Sylvester system (1.1) has no non-trivial (Φ,Ψ)-bounded solution on$\mathbb{R}$, then (1.1) has a unique (Φ,Ψ)-bounded solution on$\mathbb{R}$for every Lebesgue (Φ,Ψ)-integrable function$F:\mathbb{R}\to\mathbb{R}n2$if and only there exists a $K>0$such that $\|\Phi(t)Y(t)Z*(t)\Psi*(t)P-\Psi*-1(s)Z*-1(s)Y-1(s)\Phi-1(s)\|\le Ks$for $-\infty<s<t<\infty$and$\|\Phi(t)Y(t)Z*(t)\Psi*(t)P+\Psi*-1(s)Z*-1(s)Y-1(s)\Phi-1(s)\|\le Kt$for $-\infty<t<s<\infty$The proof follows by taking $P0=0$in Theorem 3.1.Theorem 3.3: Suppose that a fundamental matrix $Y(t)$of $T'=AT$and a fundamental matrix $Z(t)$of $T'=B*T$satisfy the conditions:(i) $\|\Phi(t)Y(t)Z*(t)\Psi*(t)P-\Psi*-1(s)Z*-1(s)Y-1(s)\Phi-1(s)\|\le K$for $t>0,s>0,s>t\|\Phi(t)Y(t)Z*(t)\Psi*(t)P-\Psi*-1(s)Z*-1(s)Y-1(s)\Phi-1(s)\|\le K$for $t\le0,s<t\|\Phi(t)Y(t)Z*(t)\Psi*(t)(P0+P-)\Psi*-1(s)Z*-1(s)Y-1(s)\Phi-1(s)\|\le K$for $t\le0,s>t,s<0\|\Phi(t)Y(t)Z*(t)\Psi*(t)P+\Psi*-1(s)Z*-1(s)Y-1(s)\Phi-1(s)\|\le K$for $t\le0,s>0,s\ge t$(ii) $\lim t\to\infty\|\Phi(t)Y(t)Z*(t)\Psi*(t)P0\|=0\lim t\to\infty\|\Phi(t)Y(t)Z*(t)\Psi*(t)P1\|=0\lim t\to\infty\|\Phi(t)Y(t)Z*(t)\Psi*(t)P-\|=0$and(iii) the function $F:\mathbb{R}\to\mathbb{R}n2$is Lebesgue-delta integrable on$\mathbb{R}$.Then, every (Φ,Ψ)-bounded solution Tof (1.1) is such that$\lim t\to\pm\infty\|\Phi(t)Y(t)Z*(t)\Psi*(t)T(t)\|=0$. The proof is similar to that of the Theorem 3.3 in [4].

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